A Decomposition Approach for a New Test-Scenario in Complex Problem Solving

Michael Engelhart

Interdisciplinary Center for Scientific Computing (IWR)
Heidelberg University
Im Neuenheimer Feld 368, 69120 Heidelberg, Germany

Joachim Funke

Department of Psychology
Heidelberg University
Hauptstr. 47–51, 69117 Heidelberg, Germany

Sebastian Sager

Institute of Mathematical Optimization
Otto-von-Guericke Universität Magdeburg
Universitätsplatz 2, 39106 Magdeburg, Germany

Abstract

Over the last years, psychological research has increasingly used computer-supported tests, especially in the analysis of complex human decision making and problem solving. The approach is to use computer-based test scenarios and to evaluate the performance of participants and correlate it to certain attributes, such as the participant's capacity to regulate emotions. However, two important questions can only be answered with the help of modern optimization methodology. The first one considers an analysis of the exact situations and decisions that led to a bad or good overall performance of test persons. The second important question concerns performance, as the choices made by humans can only be compared to one another, but not to the optimal solution, as it is unknown in general.

Additionally, these test-scenarios have usually been defined on a trial-and-error basis, until certain characteristics became apparent. The more complex models become, the more likely it is that unforeseen and unwanted characteristics emerge in studies. To overcome this important problem, we propose to use mathematical optimization methodology not only as an analysis and training tool, but also in the design stage of the complex problem scenario.

We present a novel test scenario, the *IWR Tailorshop*, with functional relations and model parameters that have been formulated based on optimization results. We also present a tailored decomposition approach to solve the resulting mixed-integer nonlinear programs with nonconvex relaxations and show some promising results of this approach.

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1. Introduction

Modern life imposes daily decision making, often with important consequences. Illustrative examples are, e.g., politicians who decide on actions to overcome a financial crisis, medical doctors who decide on complementary chemotherapy drug delivery strategies, or entrepreneurs who decide on long-term pricing strategies for the products they offer.

The process of human decision making in such tasks is the subject of research in the field of *complex problem solving* (CPS). CPS is defined as a high-order cognitive process. In research, the performance of *participants* in clearly defined *microworlds* (or tasks) is investigated.

Email address: michael.engelhart@iwr.uni-heidelberg.de (Michael Engelhart)
URL: http://mathopt.uni-hd.de (Michael Engelhart)

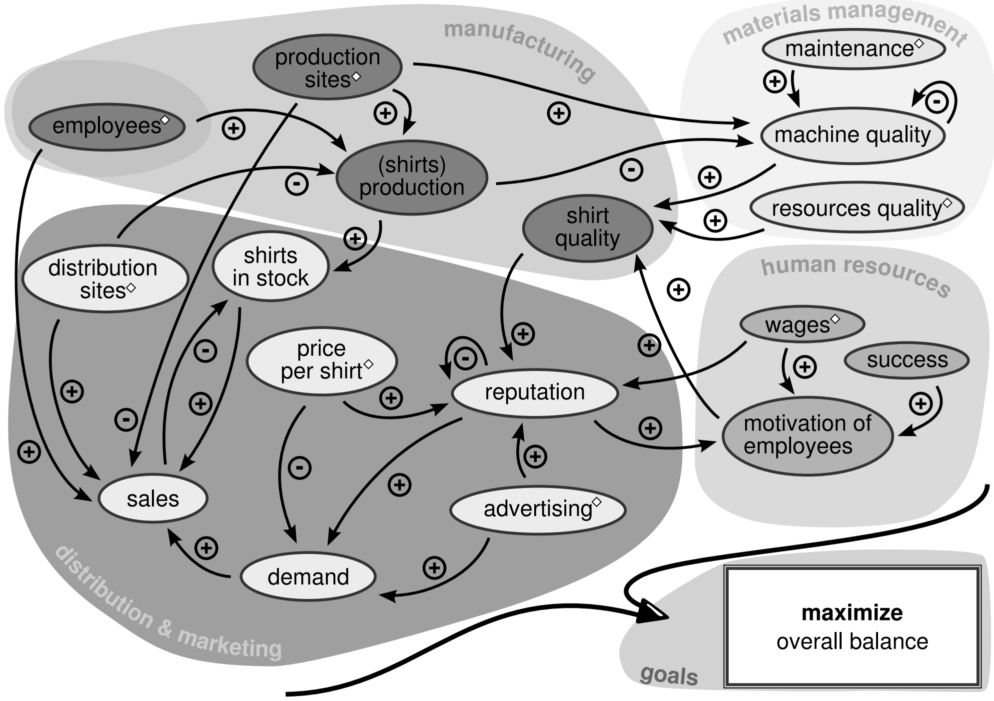

Figure 1: Schematic representation of the *IWR Tailorshop* microworld. Arrows show dependencies, the symbols (+ and -) show proportional and reciprocal influences respectively. Diamonds indicate the influence of participants' decisions.

The participant's performance is evaluated and correlated to certain attributes, such as the participant's capacity to regulate emotions.

One microworld that comprises a variety of properties such as dynamics, complexity and interdependence, discrete choices, lack of transparency, and polytely in an economical framing is the *Tailorshop*. Participants have to make economic decisions to maximize the overall balance of a small company, specialized in the production and sales of shirts. The *Tailorshop* is sometimes referred to as the "Drosophila" for CPS researchers [1] and thus a prominent example for a computer-based microworld. It has been used in a large number of studies, e.g., [2, 3, 4, 5, 6, 7]. Comprehensive reviews on studies with *Tailorshop* have been published, e.g., [8, 9, 10, 1].

The calculation of *indicator functions* to measure performance of CPS participants is by no means trivial. To measure performance within the *Tailorshop* microworld, different indicator functions have been proposed in the literature, see [11] for a recent review. In [12, 13] the question how to get a *reliable performance indicator* for the *Tailorshop* microworld has been addressed. Because all previously used indicators have unknown reliability and validity, decisions are compared to mathematically optimal solutions. For the first time a complex microworld such as *Tailorshop* has been described in terms of a mathematical model.

Therefore one can formulate the CPS task as an optimization problem. In this article, we consider dynamic scenarios with consecutive (turn-based) decisions made by participants. Such a microworld—like the *Tailorshop*—can be formulated in a general way as a *discretized mixed-integer optimal control problem* (dMIOCP)

$$\begin{array}{lllll} \max\limits_{x,u} & F(x_N) & & & \\ \text{s.t.} & x_{k+1} & = G(x_k, u_k, p, \xi), & k = n_\mathrm{s} \dots N-1, & \\ & u_{k,i} & \in \Omega_i, & k = n_\mathrm{s} \dots N-1, & \quad (1) \\ & & & i = 1 \dots n_\Omega, & \\ & 0 & \le H(x_k, u_k, p), & k = n_\mathrm{s} \dots N, & \\ & x_{n_\mathrm{s}} & = x^p_{n_\mathrm{s}}, & & \end{array}$$

for different start times $0 \le n_\mathrm{s} < N$ of the optimization and where F, G, and H are nonlinear functionals, ξ is a random variable, and Ω_i are, possibly discrete, feasible sets. State variables are denoted by x_k, scenario parameters by p, and decisions to be taken by the participants at time k by u_k. We define

$$(x^\mathrm{p}, u^\mathrm{p}) = (x^\mathrm{p}_0, \dots, x^\mathrm{p}_N, u^\mathrm{p}_0, \dots, u^\mathrm{p}_{N-1}) \qquad (2)$$

to be the vector of decisions and state variables for all months of a participant. Certain entries $x^p_{n_s}$ enter (1) as fixed initial values. Participant independent initial values $x^p_0 = x_0$ are fixed and part of the CPS microworld definition. The model is dynamic with a discrete time $k = 0 \dots N$, and N the number of turns.

Based on (1), an optimization can be performed for every turn n_s of the participant's data, starting with exactly the same conditions $x^p_{n_s}$ as the participant. The result can be used in different ways to cope with questions like how to measure performance in complex environments in an objective way and how to determine decisions which were critical for the overall performance of a participant. This technique is described in detail in [13].

Thus, the assumption that the "fruit fly of complex problem solving" is not mathematically accessible has been disproven. However, solving (1) to proven global optimality is already a challenging task. The novel methodological approach has also been combined with experimental studies, [6, 7, 13].

So far, all CPS microworlds have been developed in a purely disciplinary trial-and-error approach. To our knowledge, a systematic development of CPS microworlds based on a mathematical model, sensitivity analysis, and eventually optimization methods to choose parameters that lead to a wanted behavior of the complex system for all possible trajectories has not yet been applied. As an example for the need to do this, the mathematical modeling of the *Tailorshop* microworld in [13] led to the discovery of a priori unwanted and unrealistic winning strategies (e.g., the *vans bug*).

Therefore, in this article we present a new microworld based on the *Tailorshop*, for which optimization methods have been considered already throughout the modeling phase, the *IWR Tailorshop*. To overcome the difficulties of computing globally optimal solutions for this test-scenario, which still yields nonconvex optimization problems, we developed a decomposition approach tailored to the *IWR Tailorshop*.

Mathematical model reduction techniques are quite common in other domains, see e.g., [14, 15, 16] for an overview. The basic idea of our new approach to solve problem (1) consists of a decomposition of the MINLP into a *master* and several smaller *subproblems*. This works if the objective function is separable. The idea is related to *Lagrangian relaxation*, one of the most used relaxation strategies for *MILPs*. Its first application was the one-tree relaxation of the traveling salesman problem in the famous *Held-Karp algorithm* in [17, 18]. The traditional application fields are variants of the knapsack problem like, e.g., facility location and capacity planning [19], general assignment, network flow and the unit commitment problem [20]. The general approach is thoroughly explained in [21] and in [22]. A problem-specific decomposition approach has been proposed in [23]. The authors reformulate the MIOCP as a large-scale, structured nonlinear program (NLP) and solve a small scale linear integer program on a second level to approximate the calculated continuous aggregated output of all pumps in a water works. To obtain objective performance measures, we need guaranteed upper bounds for the maximum. Hence the mentioned techniques can not be applied in a straightforward way.

The article is organized as follows. In section 2, the *IWR Tailorshop* is introduced. Then the tailored decomposition approach is explained in section 3. We show some promising numerical results of the decomposition applied to the *IWR Tailorshop* in section 4 and conclude with an outlook in section 5.

2. The *IWR Tailorshop*-model

Based on the experience with the original *Tailorshop*-microworld described in [13] with modeling oddities, bugs, and other undesirable properties, we decided to continue our work with a mathematical model developed from scratch.

We systematically build a new microworld with desirable (mathematical) properties based on the economical framing of *Tailorshop*. These efforts lead to the new test-scenario *IWR Tailorshop*. A schematic representation of this new microworld can be found in Figure 1. Table 1 lists all states and controls the *IWR Tailorshop* contains together with corresponding units.

Compared to the *Tailorshop*, the variety of variables has been shifted towards a more abstract level. For example, the participants have no longer the task to buy or sell *machines*, but instead have to take care of the number of *production sites* x^{PS} of their company. The rather concrete variable *vans* has been replaced by more abstract *distribution sites* x^{DS}, and so on. We chose to set up *IWR Tailorshop* on such an abstract level, because this yields a more realistic position of a *decision maker* for the participants. For the majority of companies, it seems unlikely that the one who decides on the number of *employees*, the *shirt price*, and the amount of money spent for *advertising* is the same who has to ensure that enough *raw material* is bought to produce the shirts.

The mathematical representation of the *IWR Tailorshop* consists of the following set of equations for $k = n_s \dots N$, which will be explained below. Remember, that x_k denote state variables, u_k denote control variables (decision variables) and p are fixed parameters.

States	Variable	Unit	Controls	Variable	Unit
employees	x^{EM}	person(s)	shirt price	u^{SP}	M.U/shirt
production sites	x^{PS}	site(s)	advertising	u^{AD}	M.U.
distribution sites	x^{DS}	site(s)	wages	u^{WA}	M.U/person
shirts in stock	x^{SH}	shirt(s)	maintenance	u^{MA}	M.U.
production	x^{PR}	shirt(s)	resources quality	u^{RQ}	—
sales	x^{SA}	shirt(s)	recruit/dismiss employees	u^{dEM}/u^{DEM}	person(s)
demand	x^{DE}	shirt(s)	create/close production site	u^{dPS}/u^{DPS}	site(s)
reputation	x^{RE}	—	create/close distribution site	u^{dDS}/u^{DDS}	site(s)
shirts quality	x^{SQ}	—			
machine quality	x^{MQ}	—			
motivation of employees	x^{MO}	—			
capital	x^{CA}	M.U.			

Table 1: States and controls with corresponding units in the *IWR Tailorshop*. *M.U.* means monetary units.

$$x_{k+1}^{EM} = x_k^{EM} - u_k^{dEM} + u_k^{DEM} \tag{3a}$$

$$x_{k+1}^{PS} = x_k^{PS} - u_k^{dPS} + u_k^{DPS} \tag{3b}$$

$$x_{k+1}^{DS} = x_k^{DS} - u_k^{dDS} + u_k^{DDS} \tag{3c}$$

$$\begin{aligned} x_{k+1}^{DE} &= p^{DE,0} \cdot \exp\left(-p^{DE,1} \cdot u_k^{SP}\right) \\ &\quad \cdot \log\left(p^{DE,2} \cdot u_k^{AD} + 1\right) \cdot \left(x_k^{RE} + p^{DE,3}\right) \end{aligned} \tag{3d}$$

$$\begin{aligned} x_{k+1}^{RE} &= p^{RE,0} \cdot x_k^{RE} + p^{RE,1} \log\Big((p^{RE,2} \cdot u_k^{AD} \\ &\quad + p^{RE,3} \cdot u_k^{SP} \cdot (x_k^{SQ})^2 + p^{RE,4} \cdot u_k^{WA}) + 1\Big) \end{aligned} \tag{3e}$$

$$\begin{aligned} x_{k+1}^{PR} &= p^{PR,0} \cdot x_{k+1}^{PS} \\ &\quad \cdot \log\left(\frac{p^{PR,1} \cdot x_{k+1}^{EM}}{x_{k+1}^{PS} + x_{k+1}^{DS} + p^{PR,2}} + 1\right) \end{aligned} \tag{3f}$$

$$\begin{aligned} x_{k+1}^{SA} &= \min\Bigg\{p^{SA,0} \cdot x_{k+1}^{DS} \\ &\quad \cdot \log\left(\frac{p^{SA,1} \cdot x_{k+1}^{EM}}{x_{k+1}^{PS} + x_{k+1}^{DS} + p^{SA,2}} + 1\right); \\ &\quad x_k^{SH} + x_{k+1}^{PR}; p^{SA,3} \cdot x_{k+1}^{DE}\Bigg\} \end{aligned} \tag{3g}$$

$$x_{k+1}^{SH} = x_k^{SH} - x_{k+1}^{SA} + x_{k+1}^{PR} \tag{3h}$$

$$x_{k+1}^{SQ} = p^{SQ,0} \cdot x_k^{MO} + p^{SQ,1} \cdot x_k^{MQ} + p^{SQ,2} \cdot u_k^{RQ} \tag{3i}$$

$$\begin{aligned} x_{k+1}^{MQ} &= x_k^{MQ} \cdot p^{MQ,0} \cdot \exp\left(-p^{MQ,1} \frac{x_k^{PR}}{x_k^{PS} + p^{MQ,2}}\right) \\ &\quad + p^{MQ,3} \cdot \log\left(u_k^{MA} \cdot p^{MQ,4} + 1\right) \end{aligned} \tag{3j}$$

$$\begin{aligned} x_{k+1}^{MO} &= \left(1 - p^{MO,0}\right) \cdot x_k^{MO} + p^{MO,0} \\ &\quad \cdot \log\Big(p^{MO,1} \cdot u_k^{DEM} + p^{MO,2} \cdot u_k^{DPS} \\ &\qquad + p^{MO,3} \cdot u_k^{DDS} + p^{MO,4} \cdot u_k^{WA} \\ &\qquad + p^{MO,5} \cdot x_k^{RE} + p^{MO,6}\Big) \\ &\quad \cdot \exp\Big(- (p^{MO,7} \cdot u_k^{dEM} + p^{MO,8} \cdot u_k^{dPS} \\ &\qquad + p^{MO,9} \cdot u_k^{dDS}) + p^{MO,10}\Big) \cdot p^{MO,11} \end{aligned} \tag{3k}$$

$$\begin{aligned} x_{k+1}^{CA} &= p^{CA,0} \cdot \Bigg(x_k^{CA} + \left(x_{k+1}^{SA} \cdot u_k^{SP}\right) + \left(u_k^{dPS} \cdot p^{CA,1}\right) \\ &\quad + \left(u_k^{dDS} \cdot p^{CA,2}\right) - \left(x_{k+1}^{EM} \cdot u_k^{WA}\right) \\ &\quad - \left(x_{k+1}^{PR} \cdot u_k^{RQ} \cdot p^{CA,3}\right) - \left(x_k^{PS} \cdot p^{CA,4}\right) \\ &\quad - \left(x_k^{DS} \cdot p^{CA,5}\right) - u_k^{MA} - u_k^{AD} \\ &\quad - \left(x_{k+1}^{SH} \cdot p^{CA,6}\right) - \left(u^{DPS} \cdot p^{CA,7}\right) \\ &\quad - \left(u^{DDS} \cdot p^{CA,8}\right)\Bigg) \end{aligned} \tag{3l}$$

A part of these equations, (3a)-(3b), consist of a simple linear transition from month k to month $k+1$. The amount of sites created and employees recruited is added, the amount of sites closed and employees dismissed is subtracted from the inventory. Equations (3h) and (3i) are very similar to this type: the amount of shirts sold is subtracted from the current *stock*, the number of shirts produced is added. The *shirt quality* is a linear combination of three components, namely the *motivation of employees*, the *machine quality*, and the *resource quality* chosen by the participant.

The *demand* equation (3d) is more complicated and contains three factors. First, there is an exponential decrease with the *shirt price*, followed by a logarithm, which damps the influence of *advertising*. Finally, these terms are multiplied by the reputation and a certain offset. Demand here refers to the demand at this *single* company, not on the whole market.

In equation (3e), determining the *reputation*, there is a memory term consisting of a fraction of the current reputation. Additionally, there is a logarithm to dampen the effects of advertising, level of wages, and the *product value*—a product of *shirt price* and *shirt quality* to the power of two.

The *production* equation (3f) consists of a log-term, which damps the efficiency of workers per site. The assumption is, that all the employees are distributed equally over the sum of distribution and production sites. The more employees per site there are, the less productivity is yielded by one more employee, e.g., because of the limitation of space or machines. This term is multiplied by the number of production sites in compensation of the denominator in the logarithm. The *sales* equation (3g) is analog to the production equation, but with a distribution sites factor instead of production sites. Additionally, sales are limited by the number of shirts available, i.e., the sum of shirts in stock and shirts produced, and by the demand. This leads to the min-expression with three components. Note, however, that this expression can easily be transformed into inequalities by introducing a slack-variable, which is limited by all components of the minimum. This works, because the sales only have a positive effect in the objective function.

Machine quality, see equation (3j), decreases with the *load*, represented by shirts produced per production site. *Maintenance*, on the other hand, increases machine quality, damped by a logarithm again.

The *motivation* equation (3k) is a convex combination of old and new motivation levels. The level is determined by a logarithm containing positive effects (recruiting employees, creating production and distribution sites, wages, and reputation) and a negative exponential, where negative factors enter (dismissal of employees and closing production and distribution sites).

The last equation (3l), the capital, is a composition of all expenses and incomes given implicitly by the other equations: revenue per shirt, revenue per production and distribution site sold (closed), wages per employee, production costs depending on the resource quality, fixed costs for production and distribution sites, maintenance and advertising expenses, storage costs, and purchase price for production and distribution sites. The capital is subject to a certain interest rate $p^{CA,0}$.

IWR Tailorshop contains inequalities. There is a maximum storage capacity for shirts per distribution site,

$$x_k^{SH} \leq p^{SH,0} \cdot x_k^{DS}. \tag{4}$$

Recruitment depends on access to different job markets yielded by the number of sites and is limited,

$$u_k^{DEM} \leq p^{DEM,0} \cdot x_k^{PS} + p^{DEM,1} \cdot x_k^{DS}. \tag{5}$$

The overall number of sites is limited,

$$x_k^{PS} + x_k^{DS} \leq p^{tS}. \tag{6}$$

And finally, there is a limit on the sum of production sites closed within two months:

$$u_k^{dPS} + u_{k-1}^{dPS} \leq p^{dPS} \tag{7}$$

Beyond these inequalities, all states and controls except of the capital are required to be ≥ 0 and some controls have additional simple upper bounds,

$$u_k^{dEM} \leq p^{dEM}, \tag{8a}$$

$$u_k^{DPS} \leq p^{DPS}, \tag{8b}$$

$$u_k^{DDS} \leq p^{DDS}, \tag{8c}$$

$$u_k^{dDS} \leq p^{dDS}. \tag{8d}$$

Furthermore, some of the controls have to be integer,

$$u_k^{DEM}, u_k^{dEM}, u_k^{DPS}, u_k^{dPS}, u_k^{DDS}, u_k^{dDS} \in \mathbb{Z}_0^+ \tag{9}$$

and resource quality must be chosen from a finite set:

$$u_k^{RQ} \in \left\{p^{RQ,1}, \ldots, p^{RQ,n_{RQ}}\right\} \tag{10}$$

Compared to equation (1), these equations and inequalities together with the reformulation of the sales equation form the functions G and H. For the objective function F, one could easily think of different options, e.g., a weighted combination of maximizing profit, reputation, and some other factors. We decided to use the profit at the end of the discrete time-scale in this article for the sake of comparability to the original *Tailorshop*. Hence, we suggest the following objective:

$$\max_{x,u,p} \quad x_N^{CA} \tag{11}$$

Of course, the set of parameters has a significant influence on the model behavior. One could definitely dedicate a whole article on how to determine an appropriate parameter set for a microworld like *IWR Tailorshop*, depending on the aims—see also section 5 for future work regarding this issue. For this article, however, we set up a parameter set manually such that the model fulfills a certain desired behavior. The chosen parameters also yield a model behavior that makes sense for the optimization, i.e. there are feasible solutions and the optimization problem is not unbounded. The parameter values are listed in tables 2 and 3.

All these components build the *IWR Tailorshop*, which—from a mathematical point of view—is a *mixed-integer nonlinear program with nonconvex relaxation*, i.e. if the possibly discrete Ω_i in the dMIOCP (1) are replaced by some continuous $\hat{\Omega}_i \supseteq \Omega_i$, this yields a nonconvex nonlinear program. The implementation of this new model features a web-based interface and uses the widely spread *AMPL* interface [24], which allows, e.g., the use of a variety of powerful optimization algorithms.

Compared to the variants of the original *Tailorshop* microworld used in different studies, e.g. [2, 3, 4, 5, 6, 7], the dimensions of the problem are slightly smaller, but are within the same order of magnitude (e.g. 15 (9) vs. 11 (7) control variables (integer) and 16 vs. 12 state variables per month). Note, however, that first, there may be small differences between the *Tailorshop* microworlds used in former studies and that second, there are some differences between the terminology used for the variables in this article and in the articles from the psychological community (e.g. endogenous/exogenous vs. control/state). Structurally, the relation between the models is as follows. Some of the equations, such as the ones for the *shirts in stock* or the *employees*, are more or less the same or at least very similar. The main difference is, that most of the effects, for which min / max-expressions have been used in the old microworld, are modelled by smoothed terms like exp and log in *IWR Tailorshop*.

3. A Tailored Decomposition Approach

Now that we have a systematically built microworld with desirable properties, we could start doing studies with it and evaluating participants' performance based on optimal solutions as explained above and in [13]. The computation of an indicator function as described in [13], however, can only be claimed reasonably to be objective, if we can find guaranteed *globally* optimal solutions. But—as already mentioned above—the

parameter	value
$p^{SH,0}$	2000 shirts/site
$p^{DE,0}$	600.0 shirts
$p^{DE,1}$	$2 \cdot 10^{-2}$ shirts/M.U.
$p^{DE,2}$	$2 \cdot 10^{-2}$ 1/M.U.
$p^{DE,3}$	0.5
$p^{RE,0}$	0.5
$p^{RE,1}$	1.0
$p^{RE,2}$	$2.5 \cdot 10^{-5}$ 1/M.U.
$p^{RE,3}$	10^{-4} shirts/M.U.
$p^{RE,4}$	$6 \cdot 10^{-5}$ persons/M.U.
$p^{PR,0}$	99.9 shirts/sites
$p^{PR,1}$	2.0 sites/persons
$p^{PR,2}$	10^{-6} sites
$p^{SA,0}$	99.9 shirts/sites
$p^{SA,1}$	2.0 sites/persons
$p^{SA,2}$	10^{-6} sites
$p^{SA,3}$	1.0
$p^{SQ,0}$	0.2
$p^{SQ,1}$	0.3
$p^{SQ,2}$	0.5
$p^{MQ,0}$	0.8
$p^{MQ,1}$	$0.6 \cdot 10^{-2}$ sites/shirts
$p^{MQ,2}$	10^{-6} sites
$p^{MQ,3}$	0.13
$p^{MQ,4}$	0.2 M.U.$^{-1}$
$p^{MO,0}$	0.5
$p^{MO,1}$	$4 \cdot 10^{-2}$ persons^{-1}
$p^{MO,2}$	0.5 sites^{-1}
$p^{MO,3}$	0.25 sites^{-1}
$p^{MO,4}$	$2.0 \cdot 10^{-4}$ persons/M.U.
$p^{MO,5}$	0.3
$p^{MO,6}$	1.0
$p^{MO,7}$	0.7 persons^{-1}
$p^{MO,8}$	2.5 sites^{-1}
$p^{MO,9}$	2.0 sites^{-1}
$p^{MO,10}$	1.0
$p^{MO,11}$	0.5
$p^{CA,0}$	1.03
$p^{CA,1}$	5000 M.U/site
$p^{CA,2}$	3500 M.U/site
$p^{CA,3}$	5.0 M.U/shirt
$p^{CA,4}$	1000 M.U/site
$p^{CA,5}$	700 M.U/site
$p^{CA,6}$	1.5 M.U/shirt
$p^{CA,7}$	10000 M.U/site
$p^{CA,8}$	7000 M.U/site

Table 2: Parameter set for states used with *IWR Tailorshop* in this article. *M.U.* means monetary units.

parameter	**value**
n_{RQ}	4
$p^{RQ,1}$	0.25
$p^{RQ,2}$	0.5
$p^{RQ,3}$	0.75
$p^{RQ,4}$	1.0
$p^{DEM,0}$	5 persons/site
$p^{DEM,1}$	10 persons/site
p^{dEM}	10 persons
p^{DPS}	1 site
p^{dPS}	1 site
p^{DDS}	2 sites
p^{dDS}	1 site
p^{tS}	6 sites

Table 3: Parameter set for controls used with *IWR Tailorshop* in this article.

IWR Tailorshop yields a nonconvex problem. This property is unavoidable as long as we are interested in turn-based scenarios with nonlinear model equations. Hence, it is difficult to compute global solutions for such test-scenarios.

And indeed, the computation times with *Couenne 0.4* on a Intel Core i7 machine with 12 GB RAM look bad: for $N = 1$ it takes less than 1 sec, for $N = 2$ already 3 sec, and for $N = 3$ by far more than 10 min (see also Table 6). For higher values of N, we cannot hope for a solution at all before the machine runs out of memory.

The idea of the decomposition approach is now, to exploit the structure of the problem—especially the separability of the objective function, see (11)—to create a relaxation of the original problem where parts of the problem are replaced by free variables (free within some simple bounds), for which costs are computed in decoupled programs, which contain the complexity from the original program. A schematic representation of this decomposition can be found in Figure 2. The decoupling of certain parts of the original problem obviously makes the remaining *master problem* smaller and therefore easier to handle. Such a decomposition is not unique. We chose one with few overlapping variables. A schematic representation of the resulting master problem is shown in Figure 3.

The costs computation via the decoupled problems is done *offline* on a discretized grid. The decoupled problems yield themselves an optimization problem of the

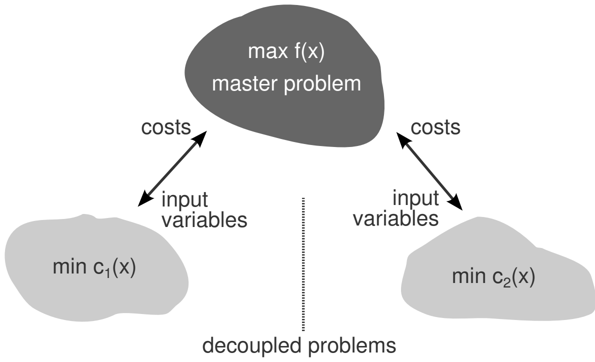

Figure 2: Schematic representation of the tailored decomposition approach.

type

min	costs
s.t.	achieve desired value of free variable (as in master problem)

The optimal solutions on the grid points can be used to fit some model, which underestimates the costs, details can be found below. This cost model is now plugged into the objective function of the master problem representing *costs* for the newly introduced free variables. We then can compute a globally optimal solution for the reduced master problem. If the relaxation is valid, this yields us a valid upper bound for the original problem. This upper bound determined by the decomposition can then be used as an indicator, how far a local solution for the original problem is away at the most from a global one.

By the decomposition, the problem size has been reduced from $12 \cdot N$ state (dependent) variables and $11 \cdot (N-1)$ control (free) variables to $4 \cdot N + 3 \cdot (N-1)$ free variables and $5 \cdot N$ states with 2 decoupled problems.

The master problem in our decomposition consists of the following equations, which form a relaxation of the original problem (3) by underestimating negative and overestimating positive effects:

$$x_{k+1}^{DE} = p^{DE,0} \cdot \exp\left(-p^{DE,1} \cdot u_k^{SP}\right) \cdot \log\left(p^{DE,2} \cdot u_k^{AD} + 1\right) \cdot \left(x_k^{RE} + p^{DE,3}\right) \tag{12a}$$

$$x_{k+1}^{RE} = p^{RE,0} \cdot x_k^{RE} + p^{RE,1} \log\left(p^{RE,2} \cdot u_k^{AD} + p^{RE,3} \cdot u_k^{SP} \cdot (u_k^{SQ})^2 + p^{RE,4} \cdot u_k^{WA} + 1\right) \tag{12b}$$

$$x^{SA}_{k+1} = \min\Big\{p^{SA,0}\cdot u^{sites}_{k+1} \cdot \log\left(\frac{p^{SA,1}\cdot u^{EM}_{k+1}}{u^{sites}_{k+1}+p^{SA,2}}+1\right); \quad x^{SH}_k + u^{PR}_{k+1}; p^{SA,3}\cdot x^{DE}_{k+1}\Big\} \tag{12c}$$

$$x^{SH}_{k+1} = x^{SH}_k - x^{SA}_{k+1} + u^{PR}_{k+1} \tag{12d}$$

$$x^{CA}_{k+1} = p^{CA,0}\cdot\Big(x^{CA}_k + \left(x^{SA}_{k+1}\cdot u^{SP}_k\right) - u^{AD}_k - u^{EM}_{k+1}\cdot u^{WA}_k - \left(x^{SH}_{k+1}\cdot p^{CA,6}\right) - f_1(u^{sites}_k; u^{PR}_k, u^{EM}_k) - f_2(u^{SQ}_k; u^{PR}_k)\Big) \tag{12e}$$

$$u^{SP}_k \in \left[lb^{SP}, ub^{SP}\right] \tag{12f}$$
$$u^{SQ}_k \in \left[lb^{SQ}, ub^{SQ}\right] \tag{12g}$$
$$u^{PR}_k \in \left[lb^{PR}, ub^{PR}\right] \tag{12h}$$
$$u^{WA}_k \in \left[lb^{WA}, ub^{WA}\right] \tag{12i}$$
$$u^{sites}_k \in \left[lb^{sites}, ub^{sites}\right] \cap \mathbb{Z}^+_0 \tag{12j}$$
$$u^{AD}_k \in \left[lb^{AD}, ub^{AD}\right] \tag{12k}$$
$$u^{EM}_k \in \left[lb^{EM}, ub^{EM}\right] \cap \mathbb{Z}^+_0 \tag{12l}$$

Here, the functions f_1 and f_2 return the costs to be determined in the decoupled problems. We choose the objective again as

$$\max_{x,u,p} \quad x^{CA}_N. \tag{13}$$

The first decoupled program, which determines the costs for a given *shirt quality*, is

$$\min \quad u^{RQ}_k \cdot \widehat{u^{PR}_{k+1}} \cdot p^{PR,cost} + u^{MA}_{k-1} \tag{14a}$$
$$\text{s.t.} \quad \widehat{u^{SQ}_k} = p^{SQ,1}\cdot x^{MQ}_k + p^{SQ,2}\cdot u^{RQ}_k \tag{14b}$$
$$x^{MQ}_k = p^{MQ,3}\cdot\log\left(p^{MQ,4}\cdot u^{MA}_{k-1}+1\right) \tag{14c}$$

$$u^{RQ}_k \in \left\{p^{RQ,1},\ldots,p^{RQ,n_{RQ}}\right\} \tag{14d}$$
$$u^{MA}_{k-1} \in \left[lb^{MA}, ub^{MA}\right] \tag{14e}$$

Here, the variables with a *hat* are considered to be given, e.g., from the *free variables* in the master problem. In the following, we call them *input variables* in this context. The second subproblem determines the costs for a given total number of *sites* and consists of the following equations.

$$\min \quad u^{DS}_{k+1}\cdot p^{CA,5} + u^{PS}_{k+1}\cdot p^{CA,4} \tag{15a}$$
$$\text{s.t.} \quad \widehat{u^{sites}_{k+1}} = u^{PS}_{k+1} + u^{DS}_{k+1} \tag{15b}$$
$$\widehat{u^{PR}_{k+1}} = p^{PR,0}\cdot\log\Big(u^{PS}_{k+1}\cdot \frac{p^{PR,1}\cdot\widehat{u^{EM}_{k+1}}}{u^{PS}_{k+1}+u^{DS}_{k+1}+p^{PR,2}}+1\Big) \tag{15c}$$

$$u^{DS}_{k+1} \in \left[lb^{DS}, ub^{DS}\right] \cap \mathbb{Z}^+_0 \tag{15d}$$
$$u^{PS}_{k+1} \in \left[lb^{PS}, ub^{PS}\right] \cap \mathbb{Z}^+_0 \tag{15e}$$

We evaluate these decoupled programs on a grid, i.e., on a discretization of the feasible interval for each *input variable*. For $u^{sites}_k \in [2, 16]$, e.g., we could choose

Original model	Decomposition
$x^{EM}_0 = 10$	$u^{EM}_0 = 10$
$x^{PS}_0 = 1$	$u^{sites}_0 = 2$
$x^{DS}_0 = 1$	
$x^{SH}_0 = 67$	$x^{SH}_0 = 67$
$x^{PR}_0 = 200$	$u^{PR}_0 = 200$
$x^{SA}_0 = 200$	$x^{SA}_0 = 200$
$x^{DE}_0 = 700$	$x^{DE}_0 = 700$
$x^{RE}_0 = 0.79$	$x^{RE}_0 = 0.79$
$x^{SQ}_0 = 0.75$	$u^{SQ}_0 = 0.75$
$x^{MQ}_0 = 0.81$	—
$x^{MO}_0 = 0.73$	—
$x^{CA}_0 = 175000$	$x^{CA}_0 = 175000$

Table 4: Initial values used for computations with original full problem and decomposition.

Original model	Decomposition
$u^{SP}_k \in [35, 55]$	$u^{SP}_k \in [35, 55]$
$u^{AD}_k \in [1000, 2000]$	$u^{AD}_k \in [1000, 2000]$
$u^{WA}_k \in [1000, 1500]$	$u^{WA}_k \in [1000, 1500]$
$u^{MA}_k \in [0, 5000]$	$u^{MA}_k \in [0, 5000]$
$x^{EM}_k \in [8, 16]$	$u^{EM}_k \in [8, 16]$
$x^{PS}_k, x^{DS}_k \in [1, 6]$	$u^{sites}_k \in [2, 6]$
$x^{PR}_k \in [0, 1000]$	$u^{PR}_k \in [0, 1000]$
$x^{SQ}_k \in [0.25, 0.75]$	$u^{SQ}_k \in [0.25, 0.75]$
$x^{SH}_k, x^{DE}_k, x^{RE}_k, x^{SA}_k \geq 0$	$x^{SH}_k, x^{DE}_k, x^{RE}_k, x^{SA}_k \geq 0$
$x^{MO}_k, x^{MQ}_k \geq 0$	—

Table 5: Simple bounds used for computations with original full problem and decomposition.

the grid $2, 4, 8, 10, 12, 14, 16$. With more than one discretized variable, this leads to multidimensional grids. For each grid point, we compute an optimal solution for the corresponding decoupled program. With the solutions for all grid points, we can fit e.g., a quadratic model, like

$$\begin{aligned} f(u_k^{SQ}; u_k^{PR}) = a_0 + a_1 \cdot u_k^{PR} + a_2 \cdot u_k^{SQ} \\ + a_3 \cdot u_k^{PR} \cdot u_k^{SQ} \\ + a_4 \cdot (u_k^{PR})^2 + a_5 \cdot (u_k^{SQ})^2. \end{aligned} \tag{16}$$

Of course, we could as well use a linear or a cubic model or something completely different. The fit can then be done by solving a simple least squares problem, with X being the set of grid points and $h(x)$ a function, which returns the optimal objective value for each grid point $x \in X$:

$$\min_{a,x} \quad \sum_{x \in X} \|f(x) - h(x)\|_2^2 \tag{17a}$$

$$\text{s.t.} \quad f(x) \leq h(x) \quad \forall x \in X. \tag{17b}$$

Especially when considering the integrality conditions, equality constraints are unlikely to be fulfilled exactly. Therefore the following reformulation is introduced for each equality constraint.

$$\widehat{u_k} = \ldots \quad \longrightarrow \quad \widehat{u_k} + \epsilon = \ldots \tag{18a}$$

$$\epsilon \in [-\rho, \rho] \tag{18b}$$

Here, ρ should be chosen reasonably small, such that the decoupled program is feasible for almost all of the grid points.

4. Numerical Results

We present first results of our decomposition approach from section 3 for the *IWR Tailorshop*. All computations have been done on an Intel Core i7 machine with 12 GB RAM running *Ubuntu 11.10 (64-bit)* with the *COIN-OR* solvers *Ipopt 3.10*, *Bonmin 1.5*, and *Couenne 0.4*. *Ipopt 3.10* is a local solver for nonlinear programs [25], which implements an *interior point method*. It is not able to treat integer constraints and has only been used for reference. *Bonmin 1.5* is a solver for general mixed-integer nonlinear programs including several algorithms [26]. For the computations in this article, *B-BB*, an NLP-based *branch-and-bound* algorithm, has been used. In contrast to these two solvers, *Couenne 0.4* is a global solver using a *spatial branch-and-bound* algorithm in order to find global optima for mixed-integer nonlinear programs with nonconvex relaxations [27]. The parameter sets used are shown in Tables 2 and 3. Initial values and simple bounds on states and controls used in all computations can be found in Tables 4 and 5.

For the decomposition, in a first step the cost functions f_1 and f_2 for the new free variables u_k^{SQ} and u_k^{sites} have been computed. Therefore the subproblems (14) and (15) have been solved on the grids

$$u_k^{SQ} \in \{0.25, 0.26, 0.27, \ldots, 0.74, 0.75\}, \tag{19a}$$

$$u_k^{PR} \in \{100, 200, 300, \ldots, 900, 1000\}, \tag{19b}$$

respectively

$$u_k^{sites} \in \{2, 3, 4, 5, 6\}, \tag{20a}$$

$$u_k^{EM} \in \{8, 9, 10, \ldots, 15, 16\}, \tag{20b}$$

$$u_k^{PR} \in \{100, 200, 300, \ldots, 900, 1000\}. \tag{20c}$$

By solving the corresponding problems of type (17) with this data, we received the following underestimators for the costs:

$$\begin{aligned} f_1(u_k^{sites}; u_k^{EM}, u_k^{PR}) = 21.6754 \\ - 944.6455 \cdot u_k^{sites} \\ + 1.4968 \cdot u_k^{PR} \\ - 28.9341 \cdot u_k^{EM} \\ + 0.1338 \cdot u_k^{sites} \cdot u_k^{PR} \\ - 3.3626 \cdot u_k^{sites} \cdot u_k^{EM} \\ - 0.0586 \cdot u_k^{PR} \cdot u_k^{EM} \\ - 1.3478 \cdot (u_k^{sites})^2 \\ + 1.8831 \cdot (u_k^{EM})^2 \end{aligned} \tag{21a}$$

$$\begin{aligned} f_2(u_k^{SQ}; u_k^{PR}) = -898.0761 + 0.1991 \cdot u_{k+1}^{PR} \\ + 4726.3749 \cdot u_{k+1}^{SQ} \\ - 8.5390 \cdot u_{k+1}^{PR} \cdot u_{k+1}^{SQ} \\ + 0.0004 \cdot (u_{k+1}^{PR})^2 \\ - 5501.7182 \cdot (u_{k+1}^{SQ})^2 \end{aligned} \tag{21b}$$

The problems for all grid points of one subproblem could be solved in less than 1 min including the fit of the quadratic model. A plot of the resulting cost function for the u^{SQ}-subproblem can be found in Figure 4. However, it was necessary to use the *global* solver *Couenne 0.4* at least in this subproblem, as we got different solutions with *Ipopt 3.10* for a relaxed version of this subproblem which obviously are not globally optimal as

N	Original model			Decomposition
	Ipopt	**Bonmin**	**Couenne**	**Couenne**
1	$\ll 1\,\mathrm{s}$	$< 1\,\mathrm{s}$	$< 1\,\mathrm{s}$	$< 1\,\mathrm{s}$
2	$\ll 1\,\mathrm{s}$	4 s	3 s	1 s
3	$< 1\,\mathrm{s}$	45 s	$> 10\,\mathrm{min}$	2 s
4	$< 1\,\mathrm{s}$	537 s	$> 10\,\mathrm{min}$	3 s
5	$< 1\,\mathrm{s}$	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	5 s
6	$< 1\,\mathrm{s}$	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	10 s
7	1 s	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	17 s
8	$< 1\,\mathrm{s}$	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	27 s
9	$< 1\,\mathrm{s}$	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	52 s
10	1 s	$> 10\,\mathrm{min}$	$> 10\,\mathrm{min}$	88 s

Table 6: Comparison of computation times between *Ipopt 3.10*, *Bonmin 1.5*, and *Couenne 0.4* for the original problem, as well as *Couenne 0.4* for the decomposition.

one can observe from the comparison to the solutions of *Couenne 0.4* in Figure 5. For the u^{sites}-subproblem a plot of the cost function is not possible due to its dimensions.

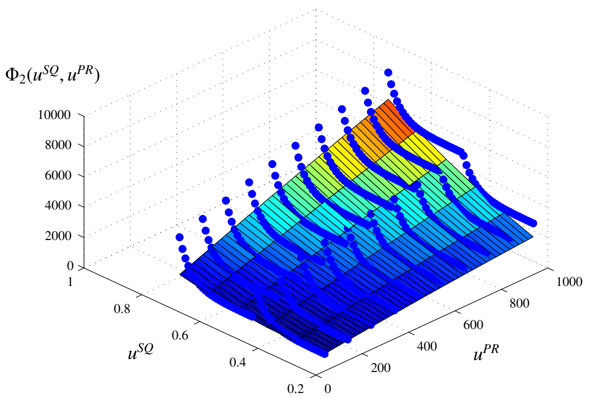

Figure 4: Cost values Φ_2 (blue dots) for solutions by *Couenne 0.4* for the decoupled problem for u^{SQ} with $p^{RQ,n_{RQ}} = 2$ on the grid $u_k^{SQ} \in \{0.25, 0.26, \ldots, 0.75\}$, $u_k^{PR} \in \{100, 200, \ldots, 1000\}$ together with the underestimating cost function (colored surface).

When comparing solutions and objective function values, three effects need to be distinguished: integrality, local vs. global solutions, and full versus overestimating reduced model. We investigated two scenarios. First, the variables u_k^{sites} respectively u_k^{PS} and u_k^{DS} have been fixed to their lower bounds 2 respectively 1. The results are listed in table 7. Here, *Ipopt 3.10* and *Bonmin 1.5* found the same solutions for the original problem, which is due to the fact that the solutions determined by *Ipopt 3.10* are already integer. Thus, there is no difference between these solvers. In this special case, *Couenne 0.4* also finds the same solutions for the original problem in an acceptable time (< 1 min). This setting allows us to focus exclusively on the third effect, the gap between our reduced and the full model. The gap determined by *Couenne 0.4* in both cases reaches from 4.0% to 16.3%.

Fortunately, this special case with fixed sites is something like a worst case. The gap is mainly due to a reduction in sales, which in turn relates to the differences between equations (3g) and (12c). Fixing the number of sites on the lower bounds results in an active first term in the minimum expressions. This is also the expression that suffers most, because the new variable u_k^{sites} is in this case twice as large as the correct expression x_k^{DS} in the original model.

If we let u_k^{sites} free within their simple bounds as shown in Table 5, the gaps between local solution to the full model and global solution to the reduced model alternate from 4.0% to 8.1%. Note that the gap relating to *Ipopt 3.10* is only for information, since *Ipopt 3.10* cannot handle integer constraints and thus solves a relaxed version of the problem. One observes that the gap first increases, but then decreases, seeming to converge to some $c > 0$. This behavior can be explained by the fact that the mentioned effect leads to an increase in cost (due to storage of not-sold shirts) that is about linear in the number of turns. The possible winnings making use of a free choice of u_k^{sites} outperforms these additional costs if the time scale for the optimization is long enough. Thus, the gap first increases and than again decreases.

N	Original model		Decomposition	Gap in %
	Ipopt	Bonmin	Couenne	
1	180995.1	180995.1	188495.0	4.0 %
2	187170.0	187170.0	198599.3	5.8 %
3	193530.2	193530.2	209006.8	7.4 %
4	200081.2	200081.2	219726.5	8.9 %
5	206828.8	206828.8	230767.7	10.4 %
6	213778.7	213778.7	242140.2	11.7 %
7	220937.2	220937.2	253853.9	13.0 %
8	228310.4	228310.4	265919.0	14.1 %
9	235904.8	235904.8	278346.0	15.2 %
10	243727.0	243727.0	291145.9	16.3 %

Table 7: Solutions using the full problem with fixed number of sites compared to the decomposition approach. Note that the solutions by *Ipopt 3.10* are already integer, so that there is no difference between *Bonmin 1.5* and *Ipopt 3.10*.

In this scenario, *Couenne 0.4* is not able anymore to find a solution for the original problem in less than 10 min for $N \geq 3$. All computation times can be found in Table 6. Obviously, the decomposition can be solved faster by orders of magnitude. Even for $N = 10$, it takes less than 2 min with *Couenne 0.4*, while *Bonmin 1.5* even is not able to compute a local solution for the original problem in less than 10 min for $N \geq 5$.

Summing up, we could estimate the gap between reduced and full model to be in the range of a few percent. We identified the most important source of gaps to be in the difference between equations (3g) and (12c). For longer time horizons and more freedom of variable choice, however, our approximation becomes better and better. The computational gains are dramatic and allow to calculate global solutions even on the full length of the time horizon.

5. Summary and Outlook

We presented a new microworld for complex problem solving, the *IWR Tailorshop*. This turn-based test-scenario yields a mixed-integer nonlinear program with nonconvex relaxation and consists of functional relations based on optimization results. With the *IWR Tailorshop* we intend to start a new era beyond trial-and-error in the definition of microworlds for analyzing human decision making.

To be able to solve the resulting problems within reasonable times, we proposed a tailored decomposition approach, where the problem is divided into a master problem and several subproblems. This decomposition is built such that it yields a valid upper bound for the corresponding global solution of the original problem and thus can be used as an indicator for the quality of local solutions of the original problem.

We finally presented promising numerical results using this decomposition approach, which indicated a high potential. In a first (worst-case like) scenario with fixed variables, the gap between decomposition and original problem was between 4.0% and 16.3%, while the original problem could also be solved to global optimality. In a second scenario, it alternated between 4.0% and 8.0%. For this scenario, only with the decomposition it was possible to get a globally optimal solution for more than 2 turns. The computation times for the decomposition are below 2 min even for 10 turns with *Couenne 0.4*, while the local solver *Bonmin 1.5* could not find a local solution for the original problem within 10 min for more than 4 turns. In future work, it could be interesting to compare these results to a Lagrangian relaxation type approach.

The parameter set used for the computations in this article has been set up manually to achieve a more or less reasonable model behavior. Here we still see high potential for improvement. For example, one could use derivative-free optimization methods to optimize the *parameter values* such that two (or even more) previously defined strategies (e.g., a high and a low price strategy) yield a similar objective value. By that, participants could follow different strategies and still perform quite well.

An important step in future work will be to collect data with participants, which will then be used to com-

N	Original model				Decomposition
	Ipopt	**Gap in %**	**Bonmin**	**Gap in %**	**Couenne**
1	181835.6	3.5%	180995.1	4.0%	188495.0
2	189161.4	4.8%	187170.0	5.8%	198599.3
3	196180.0	6.1%	193530.2	7.4%	209006.8
4	204760.9	6.8%	201860.5	8.1%	219726.5
5	215097.9	6.8%	212332.9*	8.0%	230767.7
6	226408.7	6.5%	223118.0*	7.9%	242140.2
7	239011.7	5.8%	236196.6*	7.0%	253853.9
8	252536.7	5.0%	250100.3*	6.0%	265919.0
9	266817.6	4.1%	264399.8*	5.0%	278346.0
10	281619.2	3.3%	279119.3*	4.1%	291145.9

Table 8: Solutions using the full problem compared to the decomposition approach. For solutions with a *, *Bonmin 1.5* did not find an optimal solution within 10 min. However, the gap between lower and upper bound was in all cases significantly below 1%.

pute optimal solutions for the *IWR Tailorshop* starting in states derived by the participants—as well for the original problem as for the decomposition. This will yield an indicator function with guaranteed gaps to the global solution for the original problem.

If we finally succeed to compute optimal solutions fast enough, we can take this approach even one step further: by computing the performance indicator *online*, i.e., while participants are solving the *IWR Tailorshop*, we can give an immediate feedback based on optimal solutions. It will be subject of future research how this feedback can be used to improve learning of complex problem solving competences. Answers to this question can be used to design programs to train future decision makers.

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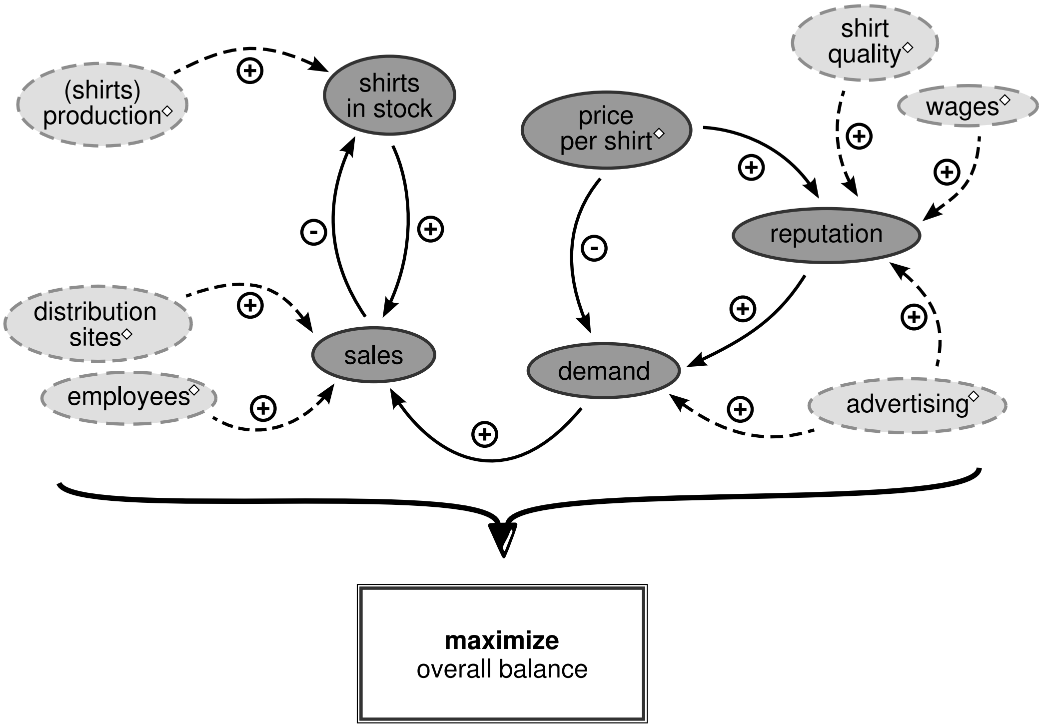
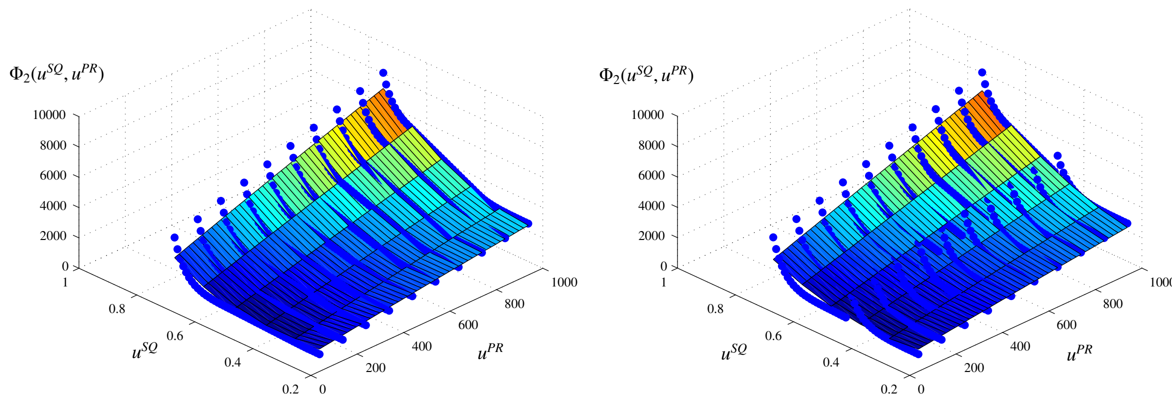


Figure 3: *IWR Tailorshop* reduced *master problem* with dependencies and proportional/reciprocal influences. Diamonds indicate free variables.



(a) Couenne (b) Ipopt

Figure 5: Cost values Φ_2 (blue dots) for solutions by *Couenne 0.4* and *Ipopt 3.10* for the decoupled problem for u^{SQ} with $p^{RQ,n_{RQ}} = 2$ and relaxed u^{RQ} on the grid $u_k^{SQ} \in \{0.25, 0.26, \ldots, 0.75\}$, $u_k^{PR} \in \{100, 200, \ldots, 1000\}$ together with the underestimating cost function (colored surface). From the differences between *Couenne 0.4* (global solver) and *Ipopt 3.10* (local solver) one can determine, that it is necessary here to use a global solver even for the decoupled problem.